Mathematical
Inequalities
& Applications
Volume 19, Number 1 (2016), 297–312
doi:10.7153/mia-19-22

LYAPUNOV–TYPE INEQUALITIES FOR THIRD–ORDER LINEAR DIFFERENTIAL EQUATIONS

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Abstract. In this paper, we obtain new Lyapunov-type inequalities for the third-order linear differential equation $x''' + q(t)x = 0$. Our work provides the sharpest results in the literature and makes corrections to those in a recently published paper [1]. Based on the above, we further establish new Lyapunov-type inequalities for more general third-order linear differential equations. Moreover, by combining these inequalities with the "uniqueness implies existence" theorems by several authors, we establish the uniqueness and hence existence-uniqueness for several classes of boundary value problems for third-order linear equations.

Mathematics subject classification (2010): 34A40, 26D10, 34B05.
Keywords and phrases: Lyapunov-type inequalities, third-order, linear differential equations, Green's functions, boundary value problems, existence and uniqueness of solutions.

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