

DIAGONAL RECURRENCE RELATIONS, INEQUALITIES, AND MONOTONICITY RELATED TO THE STIRLING NUMBERS OF THE SECOND KIND

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Abstract. In the paper, the author derives several “diagonal” recurrence relations, constructs some inequalities, finds monotonicity, and poses a conjecture related to the Stirling numbers of the second kind.

Mathematics subject classification (2010): Primary 11B73; Secondary 05A18, 11B83, 11R33, 26D15, 33B10, 44A10.

Keywords and phrases: Stirling number of the second kind, diagonal recurrence relation, inequality, logarithmically convex sequence, monotonicity, Faà di Bruno formula, Bell polynomials of the second kind, conjecture.

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