

LYAPUNOV INEQUALITY FOR FRACTIONAL DIFFERENTIAL EQUATIONS WITH PRABHAKAR DERIVATIVE

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Abstract. In this paper, we consider a fractional boundary value problem including the Prabhakar fractional derivative. We obtain associated Green function for this fractional boundary value problem and get a Lyapunov-type inequality for it.

Mathematics subject classification (2010): 26D10, 33E12, 34A08.

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