

OPERATOR VERSIONS OF SHANNON TYPE INEQUALITY

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Abstract. In this paper, we present some refinements and precise estimations of parametric extensions of Shannon inequality and its reverse one given by Furuta in Hilbert space operators. We also demonstrate an extension of operator Shannon type inequality.

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