

## ALMOST EVERYWHERE CONVERGENCE OF DYADIC TRIANGULAR–FEJÉR MEANS OF TWO–DIMENSIONAL WALSH–FOURIER SERIES

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*Abstract.* It is proved that the maximal operators of the dyadic triangular-Fejér means of two-dimensional Walsh-Fourier series is of weak type  $(1,1)$ . Moreover, the dyadic triangular-Fejér means of the function  $f \in L_1$  converge almost everywhere to  $f$  as  $n \rightarrow \infty$ .

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