

L_p -MIXED INTERSECTION BODIES

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Abstract. In this paper, we prove a dual Bergstrom type inequality for q -dual mixed volumes. Furthermore, we introduce the L_p -mixed intersection bodies for any real number $p \neq 0$, and establish some interesting inequalities for L_p -mixed intersection bodies involving a Minkowski type inequality and three Brunn-Minkowski type inequalities.

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