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A PROOF OF THE GREEN–OSHER INEQUALITY

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Abstract. In this paper we give a different proof of the Green-Osher inequality and show that equality holds if and only if γ is a circle when $F(x)$ is a strictly convex function on $(0,+\infty)$.

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