

## UNCERTAINTY PRINCIPLES FOR ORTHONORMAL SEQUENCES RELATED TO LAGUERRE HYPERGROU

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*Abstract.* In this paper, we deal with Laguerre hypergroup  $\mathbb{K} = [0, +\infty) \times \mathbb{R}$ . We prove an analogous of a time-frequency localization theorem for orthonormal sequences in  $L^2(\mathbb{K})$ . As consequence we obtain an analogous of Shapiro's Umbrella theorem. Also, we provide a mean dispersion inequality. Finally, we get a strong version of the uncertainty inequality for orthonormal bases of  $L^2(\mathbb{K})$ .

*Mathematics subject classification (2010):* 42B10, 42C25.

*Keywords and phrases:* Uncertainty inequality, Laguerre hypergroup, time-frequency localization theorem, mean dispersion inequality.

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