INEQUALITIES FOR AVERAGES OF QUASICONVEX AND SUPERQUADRATIC FUNCTIONS

S. ABRAMOVICH AND L.-E. PERSSON

Abstract. For \( n \in \mathbb{Z}^+ \) we consider the difference

\[
B_{n-1}(f) - B_n(f) := \frac{1}{a_n} \sum_{i=0}^{n-1} f \left( \frac{a_i}{a_{n-1}} \right) - \frac{1}{a_{n+1}} \sum_{i=0}^{n} f \left( \frac{a_i}{a_n} \right)
\]

where the sequences \( \{a_i\} \) and \( \{a_i - a_{i-1}\} \) are increasing. Some lower bounds are derived when \( f \) is 1-quasiconvex and when \( f \) is a closely related superquadratic function. In particular, by using some fairly new results concerning the so called "Jensen gap", these bounds can be compared. Some applications and related results about

\[
A_{n+1}(f) - A_n(f) := \frac{1}{a_n} \sum_{i=1}^{n} f \left( \frac{a_i}{a_{n+1}} \right) - \frac{1}{a_{n-1}} \sum_{i=1}^{n-1} f \left( \frac{a_i}{a_n} \right)
\]

are also included.


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REFERENCES


