

HAUSDORFF OPERATORS ON THE WEIGHTED HERZ-TYPE HARDY SPACES

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Abstract. In this paper, we study the high-dimensional Hausdorff operators on the weighted Herz-type Hardy spaces and obtain some substantial extensions from the previous results in [3]. Particularly, for the Hausdorff operators, we establish their sharp boundedness on the power weighted Herz-type Hardy spaces. Our results reveal that the Hausdorff operators have better performance in the Herz-type Hardy spaces $HK_q^{\alpha,p}(\mathbf{R}^n; w)$ ($h\dot{K}_q^{\alpha,p}(\mathbf{R}^n; w)$) than their performance in the Hardy spaces $H^p(\mathbf{R}^n; w)$ ($h^p(\mathbf{R}^n; w)$) when $0 < p < 1$.

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