

## CONCAVITY OF THE ERROR FUNCTION WITH RESPECT TO HÖLDER MEANS

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*Abstract.* In this paper, we present a necessary and sufficient condition for the concavity of the error function with respect to Hölder means.

*Mathematics subject classification* (2010): 33B20, 26E60.

*Keywords and phrases:* Error function, concavity, Hölder mean.

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