

LIMITING ULTRASYMMETRIC SEQUENCE SPACES

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Abstract. The paper gives an analytic characterization of a class of ultrasymmetric sequence spaces that are very close to ℓ_∞ . We introduce discrete descriptions of limiting J and K interpolation methods, and we apply the results and the techniques developed to the study of limiting approximation spaces.

Mathematics subject classification (2010): 46B70, 46E30, 47B38.

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