

HARDY–LITTLEWOOD–PÓLYA INEQUALITIES AND HAUSDORFF OPERATORS ON BLOCK SPACES

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Abstract. We establish the Hardy–Littlewood–Pólya inequality, the Hardy inequality and the Hilbert inequality on block spaces. Furthermore, we also have the boundedness of the Hausdorff operators on block spaces.

Mathematics subject classification (2010): 26D10, 26D15, 42B35, 46E30.

Keywords and phrases: Hardy–Littlewood–Pólya inequality, Hausdorff operator, block space, Morrey space.

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