TURÁN TYPE INEQUALITIES FOR GENERAL BESSEL FUNCTIONS

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Abstract. In this paper some Turán type inequalities for the general Bessel function, monotonicity and bounds for its logarithmic derivative are derived. Moreover we find the series representation and the relative extrema of the Turánian of general Bessel functions. The key tools in the proofs are the recurrence relations together with some asymptotic relations for Bessel functions.

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