STRICTLY POSITIVE DEFINITE KERNELS ON COMPACT TWO–POINT HOMOGENEOUS SPACES

V. S. BARBOSA AND V. A. MENEGATTO

Abstract. We present a necessary and sufficient condition for the strict positive definiteness of a real, continuous, isotropic and positive definite kernel on a compact two-point homogeneous space. The characterization is achieved using special limit formulas for Jacobi polynomials and antipodal manifolds attached to points in the homogeneous spaces. The characterization recovers that one presented in D. Chen et al. (2003) in the case in which the space is a sphere of dimension at least 2, adds to that in Menegatto et al. (2006) in the case in which the space is the unit circle and that in Beatson and zu Castell (2011) in the case of a real projective space. As an application, we use the characterization to improve upon a recent result on the differentiability of positive definite kernels on the spaces.

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