SOME INEQUALITIES INVOLVING OPERATOR MONOTONE FUNCTIONS AND OPERATOR MEANS

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Abstract. In this paper we show that if \( f : [0, \infty) \to [0, \infty) \) is an operator monotone function and \( A, B \) are positive operators such that \( 0 < pA \leq B \leq qA \), then for all \( \alpha \in [0, 1] \)
\[
f(A)^{\sharp_{\alpha}}f(B) \leq \max\{S(p), S(q)\}f(A^{\sharp_{\alpha}}B),
\]
where \( S(t) \) is the so called Specht’s ratio, and \( z_{\alpha} \) is \( \alpha \)-geometric mean.
Moreover, we present some majorization and norm inequalities for operator monotone functions. Operator monotone decreasing functions are also discussed.

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REFERENCES