

SOME INEQUALITIES INVOLVING OPERATOR MONOTONE FUNCTIONS AND OPERATOR MEANS

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Abstract. In this paper we show that if $f : [0, \infty) \rightarrow [0, \infty)$ is an operator monotone function and A, B are positive operators such that $0 < pA \leq B \leq qA$, then for all $\alpha \in [0, 1]$

$$f(A)\sharp_{\alpha}f(B) \leq \max\{S(p), S(q)\}f(A\sharp_{\alpha}B),$$

where $S(t)$ is the so called Specht's ratio, and \sharp_{α} is α -geometric mean.

Moreover, we present some majorization and norm inequalities for operator monotone functions. Operator monotone decreasing functions are also discussed.

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