Mathematical
Inequalities
& Applications
Volume 19, Number 2 (2016), 765–768 doi:10.7153/mia-19-56

AN INEQUALITY FOR t–GEOMETRIC MEANS

DINH TRUNG HOA

Abstract. Let A_i, B_i $(i = 1, \dots, m)$ be positive definite matrices, $r \geqslant 1$, $t \in [0,1]$ and $s > 0$. Then for any unitarily invariant norm $|||\cdot|||$

$$|||\sum_{i=1}^{m}(A_i \sharp_t B_i)^r||| \leqslant |||((\sum_{i=1}^{m} B_i)^{rts/2}(\sum_{i=1}^{m} A_i)^{(1-t)rs}(\sum_{i=1}^{m} B_i)^{rts/2})^{1/s}|||$$
$$\leqslant |||((\sum_{i=1}^{m} A_i)^{(1-t)rs/2}(\sum_{i=1}^{m} B_i)^{rts/2})^{1/s}|||.$$

A recent result of Audenaert [2] immediately follows from the above inequalities.

Mathematics subject classification (2010): 15A45, 15B48, 53C35.

Keywords and phrases: t-geometric mean, positive definite matrices, log-majorization, unitarily invariant norms.

REFERENCES

[1] T. ANDO, F. HIAI, *Log majorization and complementary Golden-Thompson type inequalities*, Linear Algebra Appl., **197/198** (1994), 113–131.
[2] K. M. R. AUDENAERT, *A norm inequality for pairs of commuting positive semidefinite matrices*, Electron. J. Linear Algebra, **30** (2015), 80–84.
[3] R. BHATIA, *Matrix Analysis*, Springer, New York, 1997.
[4] J. C. BOURIN, M. UCHIYAMA, *A matrix subadditivity inequality for $f(A+B)$ and $f(A)+f(B)$*, Linear Algebra Appl., **423** (2007), 512–518.
[5] J. C. BOURIN, *Matrix subadditivity inequalities and block-matrices*, Internat. J. Math. **20** (2009), 679–691.
[6] S. HAYAJNEH, F. KITTANEH, *Trace inequalities and a question of Bourin*, Bull. Aust. Math. Soc. **88** (2013), 384–389.
[7] M. LIN, *Remarks on two recent results of Audenaert*, Linear Algebra Appl., **489** (2016), 24–29.
[8] DINH TRUNG HOA, TIN-YAU TAM, *Geometry and Inequalities of Geometric Mean*, Submitted, 2015.

Mathematical Inequalities & Applications
www.ele-math.com
mia@ele-math.com