AN INEQUALITY FOR $t$–GEOMETRIC MEANS

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Abstract. Let $A_i, B_i$ ($i = 1, \ldots, m$) be positive definite matrices, $r \geq 1$, $t \in [0,1]$ and $s > 0$. Then for any unitarily invariant norm $\| \cdot \|$

$$\| \sum_{i=1}^{m} (A_i \#_t B_i)^r \| \leq \| (\sum_{i=1}^{m} A_i)^{rt^s/2} (\sum_{i=1}^{m} B_i)^{rt^s/2} )^{1/s} \|$$

$$\leq \| (\sum_{i=1}^{m} A_i)^{(1-t)rt^s/2} (\sum_{i=1}^{m} B_i)^{rt^s/2} )^{1/s} \|.$$

A recent result of Audenaert [2] immediately follows from the above inequalities.


Keywords and phrases: $t$-geometric mean, positive definite matrices, log-majorization, unitarily invariant norms.

REFERENCES


