Mathematical
Inequalities
& Applications
Volume 19, Number 3 (2016), 781–803
doi:10.7153/mia-19-58

MULTIDIMENSIONAL EXTENSIONS OF PÓLYA–KNOPP–TYPE INEQUALITIES OVER SPHERICAL CONES

CHANG-PAO CHEN, JIN-WEN LAN AND DAH-CHIN LUOR

Abstract. In this paper, we introduce a new type of limit process to evaluate the modular-type operator norm of an integral operator. This leads us to get multidimensional extensions of Pólya-Knopp-type inequalities with general measures. Our results not only extend Levin-Cochran-Lee-type inequalities from $n = 1$ to general n, but also improve the estimates given there. Moreover, they generalize Carleson's result, which is involved in the proof of Carleman's inequality. Besides these, the Pólya-Knopp-type inequalities for the cases of Laplace transform and generalized Riemann-Liouville operators are derived. For the lower bounds, a parallel theory to the above is also established.

Mathematics subject classification (2010): 47A30, 26D10, 26D15.

Keywords and phrases: Operator norm, integral operator, multidimensional modular inequalities, Hardy-Knopp-type inequalities, Pólya-Knopp-type inequalities.

REFERENCES

[1] K. F. ANDERSEN, H. P. HEINIG, *Weighted norm inequalities for certain integral operators*, Siam J. Math. Anal. **14**, no. 4, 834–844 (1983).

[2] L. CARLESON, *A proof of an inequality of Carleman*, Proc. Amer. Math. Soc. **5**, 932–933 (1954).

[3] C.-P. CHEN, J.-W. LAN, D.-C. LUOR, *The best constants for multidimensional modular inequalities over spherical cones*, Linear and Multilinear Algebra, **62**, Issue 5, pp. 683–713 (2014). DOI:10.1080/03081087.2013.777438.

[4] M. CHRIST, L. GRAFAKOS *Best constants for two nonconvolution inequalities*, Proc. Amer. Math. Soc. **123**, no. 6, 1687–1693 (1995).

[5] A. ČIŽMEŠIJA, J. PEČARIĆ, *Some new generalisations of inequalities of Hardy and Levin-Cochran-Lee*, Bull. Austral. Math. Soc. **63**, 105–113 (2001).

[6] A. ČIŽMEŠIJA, J. PEČARIĆ, I. PERIĆ, *Mixed means and inequalities of Hardy and Levin-Cochran-Lee type for multidimensional balls*, Proc. Amer. Math. Soc., **128**, no. 9, 2543–2552 (2000).

[7] A. ČIŽMEŠIJA, J. PEČARIĆ, L.-E. PERSSON, *On strengthened Hardy and Pólya-Knopp's inequalities*, J. Approx. Theory **125**, 74–84 (2003).

[8] J. A. COCHRAN, C.-S. LEE, *Inequalities related to Hardy's and Heinig's*, Math. Proc. Cambridge Philos. Soc. **96**, 1–7 (1984).

[9] P. DRÁBEK, H. P. HEINIG, A. KUNFER, *Higher dimensional Hardy inequality*, Intenat. Ser. Numer. Math. **123**, 3–16 (1997).

[10] B. GUPTA, P. JAIN, L. E. PERSSON, A. WEDESTIG, *Weighted geometric mean inequalities over cones in* $\mathbb{R}^N$, J. Inequal. Pure Appl. Math. **4**, Issue 4, Article 68 (2003).

[11] G. H. HARDY, J. E. LITTLEWOOD, G. PÓLYA, *Inequalities*, 2nd edition, Cambridge University Press, Cambridge (1967).

[12] H. P. HEINIG, *Weighted norm inequalities for certain integral operators II*, Proc. Amer. Math. Soc. **95**, no. 3, 387–395 (1985).

[13] P. JAIN, L.-E. PERSSON, A. WEDESTIG, *Carleman-Knopp type inequalities via Hardy inequalities*, Math. Inequal. Appl. **4(3)**, 343–355 (2001).

[14] A. KUFNER, L.-E. PERSSON, *Weighted inequalities of Hardy type*, World Scientific Publishing Co., Singapore, New Jersey, London, Hong Kong (2003).

Paper MIA-19-58

[15] V. I. LEVIN, S. B. STEČKIN, *Inequalities*, Amer. Math. Soc. Transl. (2), **14**, 1–29 (1960).
[16] N. LEVINSON, *Generalizations of an inequality of Hardy*, Duke Math. J. **31**, 389–394 (1964).
[17] E. R. LOVE *Inequalities related to those of Hardy and of Cochran and Lee*, Math. Proc. Cambridge Philos. Soc. **99**, 395–408 (1986).
[18] V. M. MANAKOV, *On the best constant in weighted inequalities for Riemann-Liouville integrals*, Bull. London Math. Soc. **24**, 442–448 (1992).
[19] M. NASSYROVA, L.-E. PERSSON, V. D. STEPANOV, *On weighted inequalities with geometric mean operator generated by the Hardy-type integral transform*, J. Inequal. Pure Appl. Math. **3**, Issue 4, Article 48 (electronic)(2002).
[20] L.-E. PERSSON, V. D. STEPANOV, *Weighted integral inequalities with the geometric mean operator*, J. Inequal. Appl. **7**, No. 5, 727–746 (2002).
[21] D. V. PROKHOROV *Weighted Hardy's inequalities for negative indices*, Publications Matematiques, **48**, 423–443 (2004).
[22] G. SINNAMON, *One-dimensional Hardy-type inequalities in many dimensions*, Proc. Roy. Soc. Edinburgh, **128A**, 833–848 (1998).
[23] A. WEDESTIG, *Weighted Inequalities of Hardy-type and their Limiting Iinequalities*, PhD thesis 2003:17, Luleå University of Technology, Luleå 2003.
[24] R. L. WHEEDEN, A. ZYGMUND, *Measure and Integral*, Marcel Dekker Inc., New York (1977).