

## NEW REFINEMENTS OF GENERALIZED HÖLDER'S INEQUALITY AND THEIR APPLICATIONS

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*Abstract.* In this paper, we present a series of sharpened versions of generalized Hölder's inequality. As an application in information theory, we give a new refinement of Singh's inequality with respect to the 'useful' information of order  $\alpha$  for the power distribution. The Singh's inequality include Shannon's inequality as a special case.

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