Mathematical
Inequalities
& Applications
Volume 19, Number 3 (2016), 853–862
doi:10.7153/mia-19-62

SOME CLASSES OF COMPLETELY MONOTONIC FUNCTIONS RELATED TO q–GAMMA AND q–DIGAMMA FUNCTIONS

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Abstract. In the paper, some classes of completely monotonic functions involving the q-gamma and q-digamma functions are derived. The monotonicity properties of these functions are exploited to establish a double inequality for the ratio of the q-gamma function and a double inequality for the q-digamma function. Moreover, a class of inequalities for the q-polygamma functions is presented.

Mathematics subject classification (2010): 33D05, 26D15, 26A48.
Keywords and phrases: Completely monotonic functions, logarithmically completely monotonic functions, inequalities, q-gamma function, q-digamma function.

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Paper MIA-19-62

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