

ROUGH SINGULAR INTEGRAL OPERATORS AND ITS COMMUTATORS ON GENERALIZED WEIGHTED MORREY SPACES

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Abstract. Let $\Omega \in L_q(S^{n-1})$ be a homogeneous function of degree zero with $q > 1$ and have a mean value zero on S^{n-1} . In this paper, we study the boundedness of the singular integral operators with rough kernels T_Ω and their commutators $[b, T_\Omega]$ on generalized weighted Morrey spaces $M_{p,\varphi}(w)$. We find the sufficient conditions on the pair (φ_1, φ_2) with $q' \leq p < \infty$, $p \neq 1$ and $w \in A_{p/q'}$ or $1 < p < q$ and $w^{1-p'} \in A_{p'/q'}$ which ensures the boundedness of the operators T_Ω from one generalized weighted Morrey space $M_{p,\varphi_1}(w)$ to another $M_{p,\varphi_2}(w)$ for $1 < p < \infty$. We find the sufficient conditions on the pair (φ_1, φ_2) with $b \in BMO(\mathbb{R}^n)$ and $q' \leq p < \infty$, $p \neq 1$, $w \in A_{p/q'}$ or $1 < p < q$, $w^{1-p'} \in A_{p'/q'}$ which ensures the boundedness of the operators $[b, T_\Omega]$ from $M_{p,\varphi_1}(w)$ to $M_{p,\varphi_2}(w)$ for $1 < p < \infty$. In all cases the conditions for the boundedness of the operators T_Ω , $[b, T_\Omega]$ are given in terms of Zygmund-type integral inequalities on (φ_1, φ_2) and w , which do not assume any assumption on monotonicity of $\varphi_1(x, r)$, $\varphi_2(x, r)$ in r .

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