

## SOME INEQUALITIES FOR FUNCTIONS HAVING AN $s$ -CONVEX DERIVATIVE OF SUPERIOR ORDER

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*Abstract.* Some Hermite-Hadamard type inequalities via fractional integration are derived for superior order differentiable functions having one derivative with  $s$ -convexity of either first kind or second kind. The  $n$ -th order cumulative behavior of the function in the neighborhood of the frontier of the definition interval is studied in case of the  $s$ -convexity of second kind, by means of fractional integration. The inequalities are as best as possible from the sharpness point of view, meaning that a sharpness class of functions is identified, for each inequality, within the functions that have one derivative that is  $s$ -affine either of first kind or of second kind.

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