Mathematical
Inequalities
& Applications
Volume 19, Number 3 (2016), 909–922
doi:10.7153/mia-19-66

MEAN VALUE BOUNDED VARIATION CONCEPT IN REAL SENSE: AN APPLICATION WITH NEW TECHNIQUES TO WEIGHTED INTEGRABILITY

SONGPING ZHOU

Abstract. In this paper, we consider numerical and trigonometric series with a very general monotonicity condition. A necessary and sufficient condition for the weighted integrability of sine and cosine series is proved generalizing a classical theorem of Boas and Heywood. We also remark here the inequality established in Lemma 2.7 does reflex some essential property of MVBV concept in real sense.

Mathematics subject classification (2010): 42A25, 42A50.
Keywords and phrases: Weighted integrability, mean value bounded variation, positivity, monotonicity.

REFERENCES

[1] JR. R. P. BOAS, *Integrability of trigonometric series*, J. Math. Oxford Ser. **3** (1952), 217–221.
[2] M. I. DYACHENKO AND S. TIKHONOV, *Convergence of trigonometric series with general monotone coefficients*, C. R. Math. Acad. Sci. Paris **345**:3 (2007), 123–126.
[3] L. FENG, V. TOTIK, S. P. ZHOU, *Trigonometric series with a generalized monotonicity condition*, Acta Math. Sinica English Ser. **30** (2014), 1289–1296.
[4] L. FENG AND S. P. ZHOU, *Trigonometric inequalities in the MVBV condition*, Math. Ineq. Appl. **18** (2015), 485–491.
[5] L. FENG AND S. P. ZHOU, *An application of MVBV condition in real sense for L^1-convergence of trigonometric series*, Acta Math. Hungar. **144** (2014), 416–424.
[6] P. HEYWOOD, *On the integrability of functions defined by trigonometric series*, Quart. J. Math. Oxford Ser. **5** (1954), 71–76.
[7] R. J. LE AND S. P. ZHOU, *A new condition for the uniform convergence of certain trigonometric series*, Acta Math. Hungar. **108** (2005), 161–169.
[8] L. LEINDLER, *On the uniform convergence and boundedness of a certain class of sine series*, Anal. Math. **27** (2001), 279–285.
[9] L. LEINDLER, *A new class of numerical sequences and its applications to sine and cosine series*, Anal. Math. **28** (2002), 279–286.
[10] S. A. TELYAKOVSKII, *Convergence of trigonometric series in the metric of L with coefficients that seldom change*, Proc. Steklov Inst. Math. **2** (1993), 353–358.
[11] S. A. TELYAKOVSKII, *Uniform convergence of trigonometric series with rarely changing coefficients*, Math. Notes **70** (2001), 553–559.
[12] S. TIKHONOV, *Trigonometric series with general monotone coefficients*, J. Math. Anal. Appl. **326** (2007), 721–735.
[13] S. TIKHONOV, *On L^1-convergence of Fourier series*, J. Math. Anal. Appl. **347** (2008), 416–427.
[14] M. Z. WANG AND S. P. ZHOU, *Applications of MVBV condition in L^1 integrability*, Acta Math. Hungar. **129** (2010), 70–80.
[15] D. S. YU AND S. P. ZHOU, *A generalization of the monotonicity condition and applications*, Acta Math. Hungar. **115** (2007), 247–267.

Paper MIA-19-66

[16] D. S. YU, P. ZHOU AND S. P. ZHOU, *On L^p integrability and convergence of trigonometric series*, Studia Math. **182** (2007), 215–226.
[17] D. S. YU, P. ZHOU AND S. P. ZHOU, *On L^1-convergence of Fourier series under MVBV condition*, Canad. Math. Bull. **52** (2009), 627–636.
[18] S. P. ZHOU, *Monotonicity Condition of Trigonometric Series: Development and Application*, Science Press, Beijing, 2012, in Chinese.
[19] S. P. ZHOU, F. J. FENG AND L. J. ZHANG, *Trigonometric series with piecewise mean value bounded variation coefficients*, Science China Math. **56** (2013), 1661–1677.
[20] S. P. ZHOU, P. ZHOU AND D. S. YU, *Ultimate generalization to monotonicity for uniform convergence of trigonometric series*, Science China Math. **53** (2010), 1853–1862, available: arXiv: math.CA/0611805 v1 27 Nov 2006.
[21] A. ZYGMUND, *Trigonometric Series*, Cambridge University Press, Cambridge, 1959.