HARDY AND RELLICH TYPE INEQUALITIES
WITH TWO WEIGHT FUNCTIONS

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Abstract. In the present paper we prove several sharp two-weight Hardy, Hardy-Poincaré, and Rellich type inequalities on the sub-Riemannian manifold $\mathbb{R}^{2n+1} = \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$ defined by the vector fields:

$$X_j = \frac{\partial}{\partial x_j} + 2ky_j|z|^{2k-2}\frac{\partial}{\partial l}, \quad Y_j = \frac{\partial}{\partial y_j} - 2kx_j|z|^{2k-2}\frac{\partial}{\partial l}, \quad j = 1,2,...,n$$

where $(z,y) = (x,y,l) \in \mathbb{R}^{2n+1}$, $|z| = (|x|^2+|y|^2)^{1/2}$ and $k \geqslant 1$.

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