CONVEXITY OF $\Gamma(x)\Gamma(1/x)$

G. J. O. JAMESON

Abstract. It is shown that $\log[\Gamma(x)\Gamma(1/x)]$ is convex. Gautschi's inequality $\Gamma(x)\Gamma(1/x) \geqslant 1$ is an immediate consequence.

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