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INTEGRAL INEQUALITIES OF KANTOROVICH AND FIEDLER TYPES FOR HADAMARD PRODUCTS OF OPERATORS

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Abstract. The scalar Kantorovich inequality is a reverse weighted arithmetic-harmonic mean inequality. In matrix case, this inequality is also a reverse version of Fiedler's inequality. In this paper, we establish several Kantorovich and Fiedler types integral inequalities involving Hadamard products of continuous fields of Hilbert space operators. Kantorovich type inequality in which the product is replaced by an operator mean is also investigated. Such inequalities include discrete inequalities as special cases. Moreover, we obtain the monotonicity of certain maps involving Hadamard products of operators. As special cases, we get some operator versions of Fiedler matrix inequality.

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