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CALDERÓN–HARDY SPACES WITH VARIABLE EXPONENTS AND THE SOLUTION OF THE EQUATION $\Delta^m F = f$ FOR $f \in H^{p(\cdot)}(\mathbb{R}^n)$

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Abstract. In this article we define the Calderón-Hardy spaces with variable exponents on $\mathbb{R}^n$, $\mathcal{H}^{p(\cdot)}_{q,\gamma}(\mathbb{R}^n)$, and we show that for $m \in \mathbb{N}$ the operator Δ^m is a bijective mapping from $\mathcal{H}^{p(\cdot)}_{q,2m}(\mathbb{R}^n)$ onto $H^{p(\cdot)}(\mathbb{R}^n)$.

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