CONTINUED FRACTION EXPRESSION OF THE MATHIEU SERIES

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Abstract. In this paper, we represent a continued fraction expression of the Mathieu series by a continued fraction formula of Ramanujan. As applications, we obtain some new bounds for the Mathieu series.

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