

INEQUALITIES FOR THE HARTLEY–FOURIER COSINE POLYCONVOLUTION

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Abstract. In this paper we present some new inequalities of Young's type and Saitoh's type for the Hartley-Fourier cosine polyconvolution. The inequalities of these types hold true for the Fourier convolution, but not for any convolutions or polyconvolutions, which are very few in the literature. We then use these inequalities to estimate the solution of certain integral equations and differential equations.

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