GENERALIZATION OF GOLDEN–THOMPSON
TYPE INEQUALITIES FOR NORMAL MATRICES

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Abstract. We survey some well-known matrix exponential formulae, with emphasis on log-majorization results, by using the compound matrix method.

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