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A COMPARISON TYPE PRINCIPLE FOR A CLASS OF QUASILINEAR ELLIPTIC SYSTEMS: CRITICAL AND SUBCRITICAL CASES

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Abstract. In this paper, we consider the following nonlinear elliptic system:

$$(P)\quad \begin{cases} -\Delta_{p(x)}u = u^{a(x)}v^{b(x)}, & x\in\Omega,\\ -\Delta_{q(x)}v = u^{c(x)}v^{e(x)}, & x\in\Omega,\\ u>0,\ v>0, \end{cases}$$

in a smooth bounded domain $\Omega\subset\mathbb{R}^N$, with different Dirichlet boundary conditions $u=\lambda$, $v=\mu$, $u=v=+\infty$ or $u=\lambda$, $v=+\infty$ on $\partial\Omega$, where $\lambda,\mu>0$. $p,q:\overline{\Omega}\to\mathbb{R}$ are continuous functions with $1<p(x)$, $q(x)<+\infty$, for $x\in\overline{\Omega}$, where $a(x)>p(x)-1$ and $e(x)>q(x)-1$, for $x\in\overline{\Omega}$. The main objective of this paper is to prove existence, nonexistence and uniqueness or multiplicity of positive solutions in both critical and subcritical cases. For this, a comparison type principle is used intensively.

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