

RECURSIVELY DEFINED REFINEMENTS OF THE INTEGRAL FORM OF JENSEN'S INEQUALITY

LÁSZLÓ HORVÁTH AND JOSIP PEČARIĆ

Abstract. In this paper we establish infinite chains of integral inequalities related to the classical Jensen's inequality by using special refinements of the discrete Jensen's inequality. As applications, we introduce and study new integral means (generalized quasi-arithmetic means), and give refinements of the left hand side of Hermite-Hadamard inequality.

Mathematics subject classification (2010): 26A51 (26D15).

Keywords and phrases: Classical and discrete Jensen's inequality, convex function, generalized quasi-arithmetic means, Hermite-Hadamard inequality.

REFERENCES

- [1] ABDALLAH EL FARISSI, *Simple proof and refinement of Hermite–Hadamard inequality*, J. Math. Inequal. **4** (2010), no. 3, 365–369.
- [2] H. BAUER, *Measure and Integration Theory*, Walter de Gruyter, Berlin, New York, 2001.
- [3] I. BRNETIĆ, C. E. M. PEARCE, J. PEČARIĆ, *Refinements of Jensen's inequality*, Tamkang J. Math. **31** (2000), no. 1, 63–69.
- [4] S. S. DRAGOMIR, *Two mappings in connection to Hadamard's inequalities*, J. Math. Anal. Appl. **167** (1992), no. 1, 49–56.
- [5] S. S. DRAGOMIR, P. AGARWAL, *Two new mappings associated with Hadamard's inequalities for convex functions*, Appl. Math. Lett. **11** (1998), no. 3, 33–38.
- [6] J. HALUŠKA, O. HUTNIK, *Some inequalities involving integral means*, Tatra Mt. Math. Publ. **35** (2007), no. 1, 131–146.
- [7] G. H. HARDY, J. E. LITTLEWOOD, G. PÓLYA, *Inequalities*, Cambridge University Press, Cambridge, 1978.
- [8] L. HORVÁTH, *Inequalities corresponding to the classical Jensen's inequality*, J. Math. Inequal. **3** (2009), no. 2, 189–200.
- [9] L. HORVÁTH, *A refinement of the integral form of Jensen's inequality*, J. Inequal. Appl. (2012) 2012:178, 19 pp.
- [10] L. HORVÁTH, J. PEČARIĆ, *A refinement of the discrete Jensen's inequality*, Math. Inequal. Appl. **14** (2011), no. 4, 777–791.
- [11] L. HORVÁTH, *A new refinement of the discrete Jensen's inequality depending on parameters*, J. Inequal. Appl. (2013) 2013:551, 16 pp.
- [12] L. HORVÁTH, *Weighted form of a recent refinement of the discrete Jensen's inequality*, Math. Inequal. Appl. **17** (2014), no. 3, 947–961.
- [13] L. HORVÁTH, *Infinite refinements of the discrete Jensen's inequality defined by recursion*, J. Math. Inequal. **9** (2015) no. 4, 1115–1132.
- [14] L. HORVÁTH, J. PEČARIĆ, *Refinements of the classical Jensen's inequality coming from refinements of the discrete Jensen's inequality*, Adv. Inequal. Appl., Vol. **2015** (2015), Art ID 8, 17 pp.
- [15] L. HORVÁTH, KHURAM ALI KHAN, J. PEČARIĆ, *Combinatorial Improvements of Jensen's Inequality*, Monographs in Inequalities **8**, Element, Zagreb, 2014.
- [16] J. L. W. V. JENSEN, *Sur les fonctions convexes et les inegalités entre les valeurs moyennes*, Acta Math. **30** (1906), 175–193.

- [17] KHURAM ALI KHAN, J. PEČARIĆ, *Mixed symmetric means related to the classical Jensen's inequality*, J. Math. Inequal. **7** (2013) no. 1, 43–62.
- [18] J. ROOIN, *A refinement of Jensen's inequality*, JIPAM. J. Inequal. Pure Appl. Math. **6** (2), Article 38 (2005).
- [19] H. SUN, B. LONG, Y. CHU, *On the generalized weighted quasi-arithmetic integral mean*, Int. Journal of Math. Analysis **7** (2013) no. 41, 2039–2048.
- [20] G. S. YANG, C. S. WANG, *Some refinements of Hadamard's inequality*, Tamkang J. Math. **28** (1997), 87–92.
- [21] G. S. YANG, K. L. TSENG, *On certain integral inequalities related to Hermite-Hadamard inequalities*, J. Math. Anal. Appl. **239** (1999), no. 1, 180–187.