

## GENERALIZATION OF POPOVICIU TYPE INEQUALITIES VIA GREEN'S FUNCTION AND FINK'S IDENTITY

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*Abstract.* We obtain some useful identities via Green's function and Fink's identity, and apply them to generalize the known Popoviciu's inequality for convex functions to higher order convex functions. Then we investigate the bounds for the identities related to the generalization of the Popoviciu inequality by using inequalities for the Čebyšev functional. Some results relating to the Grüss and Ostrowski type inequalities are also obtained. Finally, we construct new families of exponentially convex functions and Cauchy-type means by exploring at linear functionals associated with the obtained inequalities.

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