

## ON CONVERGENCE PROPERTIES OF GAMMA-STANCU OPERATORS BASED ON $q$ -INTEGERS

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*Abstract.* In this paper we introduce Stancu type generalization of Gamma operators based on the concept of  $q$ -integers. We first establish local approximation theorems for these operators. Next, we investigate the weighted approximation properties and give an estimate for the rate of convergence using classical modulus of continuity. Lastly, we obtain a Voronovskaya type theorem.

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