SOME INEQUALITIES FOR POWERS OF POSITIVE DEFINITE MATRICES

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Abstract. We give several matrix versions of the inequalities $a^b + b^a > 1$ and $a^a > e^{-e^{-1}}$ for positive scalars $a$ and $b$. For instance, for all positive definite matrices $A, B$, any Hermitian matrix $X$, and any unitarily invariant norm,

$$\|A^bX + XB^a\| \geq \|X\|,$$

where $a$ and $b$ are the smallest eigenvalues of $A$ and $B$, respectively.


Keywords and phrases: Convex function, Hermitian matrix, positive semidefinite matrix, positive definite matrix, singular value, unitarily invariant norm.

REFERENCES