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SOME INEQUALITIES FOR POWERS OF POSITIVE DEFINITE MATRICES

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Abstract. We give several matrix versions of the inequalities $a^b + b^a > 1$ and $a^a > e^{-e^{-1}}$ for positive scalars a and b. For instance, for all positive definite matrices A, B, any Hermitian matrix X, and any unitarily invariant norm,

$$\left|\left|\left|A^b X + X B^a\right|\right|\right| \geqslant |||X|||,$$

where a and b are the smallest eigenvalues of A and B, respectively.

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