

## CONVEX FUNCTIONS AND MEANS OF MATRICES

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*Abstract.* In this article, we prove that convex functions and log-convex functions obey certain general refinements that lead to several refinements and reverses of well known inequalities for matrices, including Young's inequality, Heinz inequality, the arithmetic-harmonic and the geometric-harmonic mean inequalities.

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