

WEAK AND STRONG TYPE ESTIMATES FOR THE COMMUTATORS OF HAUSDORFF OPERATORS

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Abstract. This paper addresses weak and strong type Lebesgue space estimates for the commutators of n -dimensional Hausdorff operators when the symbol functions belong to Lipschitz space. Strong type estimates for such commutators on classical Morrey spaces are established as well.

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