QUALITATIVE UNCERTAINTY PRINCIPLES FOR THE GENERALIZED HARTLEY TRANSFORM

HATEM MEJJAOLI

Abstract. We consider a new differential-difference operator Λ_s on the real line. We study the harmonic analysis associated with this operator. Next, we prove various mathematical aspects of the qualitative uncertainty principles, including Hardy's, Morgan's, Cowling-Price's and its variants, Beurling's, Gelfand-Shilov's, Miyachi's theorems for the generalized Hartley transform associated to the operator Λ_s.

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