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CONVOLUTION PRESERVES PARTIAL SYNCHRONICITY OF LOG–CONCAVE SEQUENCES

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Abstract. In a recent proof of the log-concavity of genus polynomials of some families of graphs, Gross et al. defined the weak synchronicity relation between log-concave sequences, and conjectured that the convolution operation by any log-concave sequence preserves weak synchronicity. In this paper we disprove it by providing a counterexample. Furthermore, we introduce the so-called partial synchronicity relation between log-concave sequences, which is proved to be (i) weaker than synchronicity, (ii) stronger than weak synchronicity, and (iii) preserved by the convolution operation.

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