

ON WEIGHTED INTEGRAL EQUATIONS WITH NEGATIVE EXPONENTS

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Abstract. This paper is concerned with the integral equation

$$u(x) = \int_{R^n} |x - y|^p k(y) u^q(y) dy,$$

where $n \geq 1$, $p \neq 0$, $q < 0$, and the weighted function $k(x)$ is smooth. This equation comes from the prescribing curvature problems. In addition, it is related to the best functions of the reversed Hardy-Littlewood-Sobolev inequality. We consider the existence and the estimates of increasing rates of the positive entire solutions in the case that $k(x)$ is bounded and unbounded respectively.

Mathematics subject classification (2010): 45E10, 45G15, 45M05, 45M20.

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