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CONVOLUTION INEQUALITIES IN WEIGHTED LORENTZ SPACES: CASE $0 < q < 1$

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Abstract. Let g be a fixed nonnegative radially decreasing kernel g. In this paper, boundedness of the convolution operator $T_g f := f * g$ between the weighted Lorentz spaces $\Gamma^q(w)$ and $\Lambda^p(v)$ is characterized in the case $0 < q < 1$. The conditions are sufficient if the kernel g is just a general measurable function.

Furthermore, the largest rearrangement-invariant (quasi-)space Y is found such that the Young-type inequality

$$\|f * g\|_{\Gamma^q(w)} \leqslant C\|f\|_{\Lambda^p(v)}\|g\|_Y$$

holds for all $f \in \Lambda^p(v)$ and $g \in Y$.

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