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EIGENVALUE INEQUALITIES RELATED TO THE ANDO–HIAI INEQUALITY

MOHAMMAD BAGHER GHAEMI AND VENUS KALEIBARY

Abstract. In this paper, we show that if f is a doubly concave function on $[0,\infty)$ and $0 < sA \leqslant B \leqslant tA$ for some scalars $0 < s \leqslant t$ with $w = t/s$, then for every $k = 1,2,\cdots,n$,

$$\lambda_k(f(A)\sharp f(B)) \leqslant \frac{w^{\frac{1}{4}} + w^{-\frac{1}{4}}}{2}\lambda_k(f(A\sharp B)),$$

where $A\sharp B = A^{\frac{1}{2}}(A^{-\frac{1}{2}}BA^{-\frac{1}{2}})^{\frac{1}{2}}A^{\frac{1}{2}}$ is the symmetric geometric mean. As an application, we give some reverses of Ando-Hiai and Golden-Thompson type inequalities. These new reverse inequalities, improve some known results.

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