

A COMPLETE 3-DIMENSIONAL BLASCHKE-SANTALÓ DIAGRAM

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Abstract. We present a complete 3-dimensional Blaschke-Santaló diagram for planar convex bodies with respect to the four classical magnitudes: inradius, circumradius, diameter, and (minimal) width in Euclidean spaces.

Mathematics subject classification (2010): Primary 52A10, 52A40; Secondary 52A20.

Keywords and phrases: Inradius, circumradius, geometric inequalities, Blaschke diagram, Blaschke-Santaló diagram, shape diagrams.

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