

NEW INEQUALITIES FOR PRODUCTS OF CROSS-SECTION MEASURES

HORST MARTINI AND ZOKHRAB MUSTAFAEV

Abstract. The main purpose of this paper is to sharpen some upper and lower bounds on products of cross-section measures of centered convex bodies. The bounds are given in terms of relative inner and outer radii of isoperimetric spaces, and improve previously known results in the symmetric case. Thus, our results mainly refer to the geometry of finite dimensional real Banach spaces.

Mathematics subject classification (2010): 46B20, 52A20, 52A21, 52A40.

Keywords and phrases: Affine isoperimetric inequalities, cross-section measures, inner radius, intersection body, isoperimetrix, Minkowski geometry, outer radius, projection body.

REFERENCES

- [1] N. S. BRANNEN, *Volumes of projection bodies*, *Mathematika* **43** (1996), 255–264.
- [2] H. BUSEMANN, *A theorem on convex bodies of the Brunn-Minkowski type*, *Proc. Nat. Acad. Sci. USA* **35** (1949), 27–31.
- [3] R. J. GARDNER, *Geometric Tomography*, second edition, *Encyclopedia of Mathematics and its Applications* **58**, Cambridge University Press, New York, 2006.
- [4] P. GOODEY AND W. WEIL, *Zonoids and generalizations*, In: *Handbook of Convex Geometry*, vol. B, pp. 1297–1326, North-Holland, Amsterdam et al., 1993.
- [5] E. LUTWAK, *Intersection bodies and dual mixed volumes*, *Adv. Math.* **71** (1988), 232–261.
- [6] E. LUTWAK, *On a conjectured projection inequality of Petty*, *Contemp. Math.* **113** (1990), 171–182.
- [7] E. LUTWAK, *Selected affine isoperimetric inequalities*, In: *Handbook of Convex Geometry*, vol. A, pp. 151–176, North-Holland, Amsterdam et al., 1993.
- [8] H. MARTINI, *On inner quermasses of convex bodies*, *Arch. Math.* **52** (1989), 402–406.
- [9] H. MARTINI AND Z. MUSTAFAEV, *Some applications of cross-section measures in Minkowski spaces*, *Period. Math. Hungar.* **53** (2006), 185–197.
- [10] H. MARTINI AND Z. MUSTAFAEV, *On unit balls and isoperimetric spaces in normed spaces*, *Colloq. Math.* **127** (2012), 133–142.
- [11] C. A. ROGERS AND G. C. SHEPHARD, *Convex bodies associated with a given convex body*, *J. London Math. Soc.* **33** (1958), 270–281.
- [12] C. SAROGLU, *Volumes of projection bodies of some classes of convex bodies*, *Mathematika* **57** (2011), 329–353.
- [13] R. SCHNEIDER, *Stability for some extremal properties of the simplex*, *J. Geom.* **96** (2009), 135–148.
- [14] R. SCHNEIDER, *Convex Bodies: The Brunn-Minkowski Theory*, second edition, *Encyclopedia of Mathematics and its Applications* **151**, Cambridge University Press, 2014.
- [15] R. SCHNEIDER AND W. WEIL, *Zonoids and related topics*, In: *Convexity and its Applications*, pp. 269–317, Birkhäuser, Basel, 1983.
- [16] J. E. SPINGARN, *An inequality for sections and projections of convex sets*, *Proc. Amer. Math. Soc.* **118** (1993), 1219–1224.
- [17] A. C. THOMPSON, *Minkowski Geometry*, *Encyclopedia of Mathematics and its Applications* **63**, Cambridge University Press, 1996.
- [18] G. TÓTH, *Notes on Schneider's stability estimates for convex sets in Minkowski spaces*, *J. Geom.* **104** (2013), 585–598.