

ON A GENERALIZATION OF A THEOREM OF LEVIN AND STEČKIN AND INEQUALITIES OF THE HERMITE–HADAMARD TYPE

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Abstract. We give new necessary and sufficient conditions for higher order convex ordering. These results generalize the Levin-Stečkin theorem (1960) on convex ordering. The obtained results can be useful in the study of the Hermite-Hadamard type inequalities and in particular inequalities between the quadrature operators.

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REFERENCES

- [1] M. BESSENYEI, *Hermite–Hadamard-type inequalities for generalized convex functions*, J. Inequal. Pure Appl. Math. **9** (2008), 1–51.
- [2] M. BESSENYEI AND Z. S. PÁLES, *Higher-order generalizations of Hadamard’s inequality*, Publ. Math. Debrecen **61** (2002), no. 3-4, 623–643.
- [3] M. BESSENYEI AND Z. S. PÁLES, *Hadamard-type inequalities for generalized convex functions*, Math. Inequal. Appl. **6**, 3 (2003), 379–392.
- [4] M. BESSENYEI AND Z. S. PÁLES, *On generalized higher-order convexity and Hermite–Hadamard-type inequalities*, Acta Sci. Math. (Szeged), **70** (2004), no. 1–2, 13–24. MR 2005e:26012.
- [5] M. BESSENYEI AND Z. S. PÁLES, *Characterization of higher-order monotonicity via integral inequalities*, Proc. R. Soc. Edinburgh Sect. A, **140A**, 1 (2010), 723–736.
- [6] H. BRASS AND K. PETRAS, *Quadrature theory. The theory of numerical integration on a compact interval*, Mathematical Surveys and Monographs **178**. American Mathematical Society, Providence, RI, 2011.
- [7] H. BRASS AND G. SCHMEISSER, *Error estimates for interpolatory quadrature formulae*, Numer. Math. **37**, 3 (1981), 371–386.
- [8] M. DENUIT, C. LEFÈVRE AND M. SHAKED, *The s -convex orders among real random variables, with applications*, Math. Inequal. Appl., **1** (1998), 585–613.
- [9] S. S. DRAGOMIR AND C. E. M. PEARCE, *Selected Topics on Hermite-Hadamard Inequalities and Applications*, RGMIA Monographs, Victoria University, 2000, (online: <http://rgmia.vu.edu.au/monographs/>).
- [10] A. FLOREA, E. PĂLTĂNEA AND D. BĂLĂ, *Convex Ordering Properties and Applications*, J. Math. Inequal **9**, 4 (2015), 1245–1257.
- [11] S. KARLIN AND A. NOVIKOFF, *Generalized convex inequalities*, Pacific J. Math. **13**, 4 (1963), 1251–1279.
- [12] M. KUCZMA, *An Introduction to the Theory of Functional Equations and Inequalities*, Prace Naukowe Uniwersytetu Śląskiego w Katowicach, vol. **489**, Państwowe Wydawnictwo Naukowe – Uniwersytet Śląski, Warszawa, Kraków, Katowice, 1985.
- [13] V. I. LEVIN AND S. B. STEČKIN, *Inequalities*, Amer. Math. Soc. Transl. **14**, 2 (1960), 1–22.
- [14] C. P. NICULESCU AND L. E. PERSSON, *Convex functions and their applications. A contemporary approach*, Springer, New York 2006.
- [15] A. OLBRYŚ, T. SZOSTOK, *Inequalities of the Hermite-Hadamard type involving numerical differentiation formulas*, Results. Math., **67** (2015), 403–416.

- [16] J. OHLIN, *On a class of measures of dispersion with application to optimal reinsurance*, ASTIN Bulletin, **5** (1969), 249–266.
- [17] T. POPOVICIU, *Sur quelques propriétés des fonctions d'une ou de deux variables réelles*, Mathematica, **8** (1934), 1–85.
- [18] T. RAJBA, *On probabilistic characterizations of convexity and delta-convexity*, Conference on Inequalities and Applications '14, September 7–13, 2014, Hajdúszoboszló (Hungary).
- [19] T. RAJBA, *New integral representations of n th order convex functions*, J. Math. Anal. Appl., **379**, 2 (2011), 736–747.
- [20] T. RAJBA, *On the Ohlin lemma for Hermite-Hadamard-Fejér type inequalities*, Math. Inequal. Appl., **17**, 2 (2014), 557–571.
- [21] T. RAJBA, *On strong delta-convexity and Hermite-Hadamard type inequalities for delta-convex functions of higher order*, Math. Inequal. Appl., **18**, 1 (2015), 267–293.
- [22] T. SZOSTOK, *Levin Stečkin theorem and inequalities of the Hermite-Hadamard type*, arXiv preprint, arXiv:1411.7708v1 [math.CA].
- [23] T. SZOSTOK, *Ohlin's lemma and some inequalities of the Hermite-Hadamard type*, Aequationes mathematicae, (2014), 1–12, August 06, 2014.
- [24] S. Z. WAŚOWICZ, *Inequalities between the quadrature operators and error bounds of quadrature rules*, J. Inequal. Pure Appl. Math., **8**, 2 (2007), Article 42, 8 pp.
- [25] S. Z. WAŚOWICZ, *On quadrature rules, inequalities and error bounds*, J. Inequal. Pure Appl. Math., **9**, 2 (2008), Article 36, 4 pp.
- [26] S. Z. WAŚOWICZ, *A new proof of some inequality connected with quadratures*, J. Inequal. Pure Appl. Math., **9**, 1 (2008), Article 7, 3 pp.
- [27] S. Z. WAŚOWICZ, *On some extremalities in the approximate integration*, Math. Inequal. Appl., **13** (2010), 165–174.