# VARIABLE INTEGRAL AND SMOOTH EXPONENT TRIEBEL–LIZORKIN SPACES ASSOCIATED WITH A NON–NEGATIVE SELF–ADJOINT OPERATOR

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*Abstract.* In this paper, variable integral and smooth exponent Triebel-Lizorkin spaces associated with a non-negative self-adjoint operator are introduced. Then equivalent norms and atomic decomposition of these new spaces are given.

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