EXPLICIT TRACES OF FUNCTIONS FROM SOBOLEV SPACES AND QUASI–OPTIMAL LINEAR INTERPOLATORS

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Abstract. Let $\Lambda \subset \mathbb{R}$ be a strictly increasing sequence. For $r = 1, 2$, we give a simple explicit expression for an equivalent norm on the trace spaces $W_p^r(\mathbb{R})|_\Lambda$, $\dot{L}_p^r(\mathbb{R})|_\Lambda$ of the non-homogeneous and homogeneous Sobolev spaces with r derivatives $W_p^r(\mathbb{R})$, $\dot{L}_p^r(\mathbb{R})$. As Fefferman, Israel and Luli show, such simple result is impossible for Sobolev spaces of $\mathbb{R}^d$ for $d \geqslant 2$.

We also construct an interpolating spline of low degree having optimal norm up to a constant factor. This spline and the equivalent trace norm are very easy to compute. We also conjecture, what is the expression for the equivalent trace norm for any $r \geqslant 1$ and give some partial results, which, in particular, confirm this conjecture.

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