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SOME NEW GENERALIZED FORMS OF HARDY'S TYPE INEQUALITY ON TIME SCALES

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Abstract. In this paper, we prove some new dynamic inequalities from which some known dynamic inequalities on time scales, some integral and discrete inequalities due to Hardy, Copson, Chow, Levinson, Pachpatte Yang and Hwang will be deduced as special cases. Also, some new corresponding integral and discrete inequalities will be formulated. The results will be proved by employing the chain rule, integration by parts formula, Hölder's inequality and Jensen's inequality on time scales.

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