

ON HARDY'S INEQUALITIES FOR THE SPECIAL HERMITE EXPANSIONS

JINSEN XIAO AND JIANXUN HE

Abstract. This article presents two types of Hardy's inequalities for the special Hermite expansions. The proofs are mainly based on an estimate of atomic functions deduced by the horizontal Taylor formula of the Heisenberg group.

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