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ON THE COMPOSITION OF FUNCTIONS IN MULTIDIMENSIONAL BESOV SPACES

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Abstract. For the composition operator $T_f : g \mapsto f \circ g$ we find a class of functions $f : \mathbb{R} \to \mathbb{R}$ for which there exists a family of positive constants $c(f,t)$, $t > 0$, such that the estimate

$$\|T_f(g)\|_{B^s_{p,q}(\mathbb{R}^n)} \leqslant c(f,t)\|g\|_{B^s_{p,q}(\mathbb{R}^n)}$$

holds, for all $g \in W^1_\infty \cap B^s_{p,q}(\mathbb{R}^n)$ satisfying $\|\nabla g\|_\infty \leqslant t$ (or $g \in L_\infty \cap B^s_{p,q}(\mathbb{R}^n)$ with $\|g\|_\infty \leqslant t$ and $[s] = 1$). We establish this assertion, for all $f \in B^{s_1,loc}_{p,\infty}(\mathbb{R})$ with $s_1 > 1 + 1/p$, in the case $1 < p < \infty$, $0 < q \leqslant \infty$ and $0 < s < s_1$.

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